Α' ΛΥΚΕΙΟΥ

ΑΛΓΕΒΡΑ

ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ 1ο

Α. Ορισμός σχολικού βιβλίου, σελ. 63.

Β. Απόδειξη, σελ. 45, σχολικού βιβλίου.

Γ. **α.** Λ

   β. Σ

   γ. Λ

   δ. Σ

   ε. Λ

ΘΕΜΑ 2ο

α.

$$D = \begin{vmatrix} \lambda + 2 & 5 \\ 1 & \lambda - 2 \end{vmatrix} = \lambda^2 - 4 - 5 = \lambda^2 - 9 = (\lambda + 3)(\lambda - 3)$$

$$D_x = \begin{vmatrix} 5 & 5 \\ -5 & \lambda - 2 \end{vmatrix} = 5(\lambda - 2) + 25 = 5\lambda - 10 + 25 = 5\lambda + 15 = 5(\lambda + 3)$$

$$D_y = \begin{vmatrix} \lambda + 2 & 5 \\ 1 & -5 \end{vmatrix} = -5(\lambda + 2) - 5 = -5\lambda - 10 - 5 = -5\lambda - 15 = -5(\lambda + 3)$$

β. Αν $D \neq 0$, τότε $(\lambda + 3)(\lambda - 3) \neq 0$, οπότε $\lambda \neq -3$ και $\lambda \neq 3$, επομένως το σύστημα έχει μοναδική λύση την:

$$x = \frac{D_x}{D} = \frac{5(\lambda + 3)}{(\lambda + 3)(\lambda - 3)} = \frac{5}{\lambda - 3}$$

$$y = \frac{D_y}{D} = \frac{-5(\lambda + 3)}{(\lambda + 3)(\lambda - 3)} = \frac{-5}{\lambda - 3}$$

Αν $D = 0 \iff \lambda = -3$ ή $\lambda = 3$

i)

$$\left.\begin{array}{l}\lambda = -3 \;\Rightarrow\; -x+5y=5 \\ x-5y=-5\end{array}\right\} \cdot(-1) \quad \left.\begin{array}{l}x-5y=-5 \\ x-5y=-5\end{array}\right\} \Leftrightarrow x-5y=-5 \Leftrightarrow$$

$$x = 5y-5, \quad y \in \Re$$

$(x, y) = (5y\text{ -}5,\ y),\ y \in \Re$, άπειρες λύσεις

ii)

$$\left.\begin{array}{l}\lambda = 3 \;\Rightarrow\; 5x+5y=5 \\ x+y=-5\end{array}\right\} :5 \quad \left.\begin{array}{l}x+y=1 \\ x+y=-5\end{array}\right\} \Leftrightarrow 1 = -5$$

$$\Leftrightarrow$$

αδύνατο

γ.

$$x_0 = \frac{5}{\lambda-3}, \qquad y_0 = \frac{-5}{\lambda-3}$$

$$\left|\frac{5}{x_o}-\frac{5}{y_o}\right|=1 \Leftrightarrow \left|\frac{5}{\frac{5}{\lambda-3}}-\frac{5}{-\frac{5}{\lambda-3}}\right|=1 \Leftrightarrow |\lambda-3+\lambda-3|=1 \Leftrightarrow$$

$$|2\lambda-6|=1 \Leftrightarrow 2\lambda-6=1 \quad \text{ή} \quad 2\lambda-6=-1 \Leftrightarrow \lambda=\frac{7}{2} \quad \text{ή} \quad \lambda=\frac{5}{2} \quad (\delta\varepsilon\kappa\tau\acute{\varepsilon}\varsigma)$$

ΘΕΜΑ 3ο

α.   Αφού η εξίσωση έχει δύο ρίζες άνισες, τότε

$$\Delta \succ 0 \Leftrightarrow (1-\lambda)^2 - 4 \succ 0 \Leftrightarrow (1-\lambda)^2 \succ 4 \Leftrightarrow |1-\lambda| \succ 2$$

β. $|1-\lambda| \succ 2 \Leftrightarrow 1-\lambda \succ 2 \quad \text{ή} \quad 1-\lambda \prec -2 \Leftrightarrow 1-2 \succ \lambda \quad \text{ή} \quad 1-2 \prec \lambda$

$$\lambda \prec -1 \quad \text{ή} \quad \lambda \succ 3$$

γ.   Από τους τύπους του Vieta

$$K = x_1 + x_2 = -\frac{\beta}{\alpha} = -\frac{1-\lambda}{1} = \lambda - 1$$

$$\Lambda = x_1 \cdot x_2 = \frac{\gamma}{\alpha} = 1$$

$$M = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 \cdot x_2} = \frac{\lambda-1}{1} = \lambda - 1$$

δ. $\lambda x_1 x_2 (x_1 + x_2) + 3(x_1 + x_2) = 5$

$\lambda \cdot 1 \cdot (\lambda - 1) + 3 \cdot (\lambda - 1) = 5$

$\lambda^2 - \lambda + 3\lambda - 3 - 5 = 0$

$\lambda^2 + 2\lambda - 8 = 0$

$\left. \begin{array}{ll} \lambda = 2 & \acute{\eta} \quad \lambda = -4 \\ \lambda \prec -1 & \acute{\eta} \quad \lambda \succ 3 \end{array} \right\} \quad \Leftrightarrow \lambda = -4 \text{, δεκτή}$

ΘΕΜΑ 4ο

α.

$\left. \begin{array}{l} f(-2) = f(4) \\ f(2) = f(-1) \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 2\alpha \cdot (-2) - 5 = 4 + \beta \\ 2 + \beta = 2\alpha(-1) - 5 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} -4\alpha - 5 = 4 + \beta \\ 2 + \beta = -2\alpha - 5 \end{array} \right\} \Leftrightarrow$

$\begin{array}{l} -4\alpha - \beta = 9 \\ \underline{2\alpha + \beta = -7} \\ \left. \begin{array}{l} -2\alpha = 2 \quad \Leftrightarrow \quad \alpha = -1 \\ 2\alpha + \beta = -7 \end{array} \right\} \Leftrightarrow 2 \cdot (-1) + \beta = -7 \Leftrightarrow \beta = -5 \end{array}$

β. $\lambda \varepsilon_1 = \lambda \varepsilon_2 \Leftrightarrow \lambda^4 + 2 = 13\lambda^2 - 34 \Leftrightarrow \lambda^4 - 13\lambda^2 + 36 = 0 \Leftrightarrow$

$\left. \begin{array}{l} (\lambda^2)^2 - 13\lambda^2 + 36 = 0 \\ \theta\acute{\epsilon}\tau\omega \quad \lambda^2 = \omega \end{array} \right\} \Leftrightarrow \omega^2 - 13\omega + 36 = 0 \Leftrightarrow \omega_1 = 4 \ \acute{\eta} \ \omega_2 = 9$

$\lambda^2 = 4 \Leftrightarrow \lambda = \pm 2$

$\lambda^2 = 9 \Leftrightarrow \lambda = \pm 3$

γ.   το πεδίο ορισμού της f είναι $A = [-5,5)$

$\bullet \ -5 \leq x \prec 2$

$f(x) = 1 \Leftrightarrow -2x - 5 = 1 \Leftrightarrow -2x = 6 \Leftrightarrow x = -3 \quad$ δεκτή

$\bullet \ 2 \leq x \prec 5$

$f(x) = 1 \Leftrightarrow x - 5 = 1 \Leftrightarrow x = 6 \quad$ απορρίπτεται